Worksheet 16

1. Let $X \sim Gamma(\alpha, \beta)$. Find $\mathbb{E}X$ using the moment generating function.

2. Let $X \sim Gamma(\alpha, \beta)$. Find $Var(X)$ using the moment generating function.

3. Let $X \sim Gamma(\alpha, \beta)$. Show that $c \cdot X$ also has a Gamma distribution and find it's parameters. Hint: Use the moment generating function.

4. Let $X \sim Gamma(\alpha, \beta)$ and $Y \sim Gamma(\theta, \beta)$ be independent random variables. Find the distribution of $Z = X + Y$.

5. Let $X \sim Gamma(\alpha, \beta)$. For a sufficently large α, (a) how and (b) why can we approximate X by a normal distribution?

6. The normalizing constant in the Beta distribution gives that the following must true for any positive α and β:

$$\left[\frac{\Gamma(\alpha)\Gamma(\beta)}{\Gamma(\alpha+\beta)}\right] = \int_0^1 \left[x^{\alpha-1}(1-x)^{\beta-1}\right] dx$$

Let $X \sim Beta(\alpha, \beta)$. Find $\mathbb{E}(X)$. Start by writing down anx integral definition of the expected value, but use the formula above to solve it. Note that you can simplify the final result without using the Gamma function.

7. Let $X \sim Beta(\alpha, \beta)$. Using the same trick as the previous question, what is $\mathbb{E}X^2$?