Worksheet 15

1. Let $X \sim N(\mu, \sigma^2)$. Show that $\mathbb{E}(X)$ is equal to μ using the moment generating function.

2. Let $X \sim N(\mu, \sigma^2)$. Show that $Var(X)$ is equal to σ^2 using the moment generating function.

3. Let $X \sim N(\mu_1, \sigma_1^2)$ and $Y \sim N(\mu_2, \sigma_2^2)$ be independent random variables. Let $W = X+Y$. Show that W, as defined above, is a normally distributed random variable. Find its mean and variance. Hint: Use the moment generating function.

4. Let $X = \mu + \sigma Z$ where $Z \sim N(0,1)$. Show that $X \sim N(\mu, \sigma^2)$. Hint: moment generating function!

5. Let $X \sim N(3,5)$. Write the probability $\mathbb{P}[X > 10]$ as a function of Φ.

6. ($\star$) Let $X \sim N(0,1)$. Show that the moment generating function $m_X(t)$ is equal to $e^{t^2/2}$. The full form on the handout follows from the other results established above.