Worksheet 14

1. Let X be a continuous random variable defined over the set $[0, b]$ for some $b > 0$ whose density function is some constant value C over that interval and 0 otherwise. Find the constant C that makes this a valid density function.

2. What are $\mathbb{E}X$ and $Var(X)$ for X as defined in question 1?

3. Let X be a continuous random variable with density $f(x) = \lambda e^{-\lambda x}$ for $x \geq 0$ and some fixed $\lambda > 0$. This is called the exponential distribution, which we can write $X \sim Exp(\lambda)$. What is the cumulative distribution $F(x)$? Find $\mathbb{P}[x \geq 1]$.

4. Let $X \sim Exp(\lambda)$ and fix two constants $a > 0$ and $b > 0$. We can define the following:

$$\mathbb{P}[X > a + b | X > a] = \frac{\mathbb{P}[(X > a + b) \cap (X > a)]}{\mathbb{P}[(X > a)]} = \frac{\mathbb{P}[X > a + b]}{\mathbb{P}[(X > a)]}$$

Calculate this quantity using the CDF of the exponential, and try to simplify the result in terms of a probability. The exponential is used to model wait times between independent events, largely due to the property you should see here.

5. Find the MGF of the exponential distribution for $t < \lambda$.

6. If $X \sim Exp(\lambda)$, find $\mathbb{E}X$ and $Var(X)$.

7. Consider working as a PA in an urgent care facility late at night. On average, you know that a patient comes in every 10 minutes. You need to go to the back room to restock the insulin in your front office, which takes 3 minutes. Using the exponential distribution, how likely is it that a patient will arrive while you are gone?

8. If the wait times between events is distributed as $Exp(\lambda)$ then the number of events that occurs in any interval of size t is given by a random variable that has a Poisson distribution with rate $t \cdot \lambda$. Using the data from the previous example, how many patients do you see on average over an 8 hour shift?