Worksheet 13

1. Let X and Y be independent random variables with the following probability mass functions:

$$p_X(x) = \begin{cases} 0.2, & \text{if } x = 1 \\ 0.5, & \text{if } x = 2 \\ 0.3, & \text{if } x = 3 \end{cases} \quad p_Y(y) = \begin{cases} 0.7, & \text{if } y = 1 \\ 0.2, & \text{if } y = 2 \\ 0.1, & \text{if } y = 3 \end{cases}$$

For X, find the expected value, the variance and the moment generating function. Sketch the cdf of X. What are the expected value of Y and the expected value of $X + Y$?

2. Let X and Y be defined as in the previous question. Let $Z = max(X, Y)$. Write the pmf and sketch the cdf of Z.[1]

[1] In general, you need to work out all nine possible combinations of X and Y. A shortcut is to realize that you don't need to figure out the mass at 3 directly since it is one minus the mass at 1 and 2.

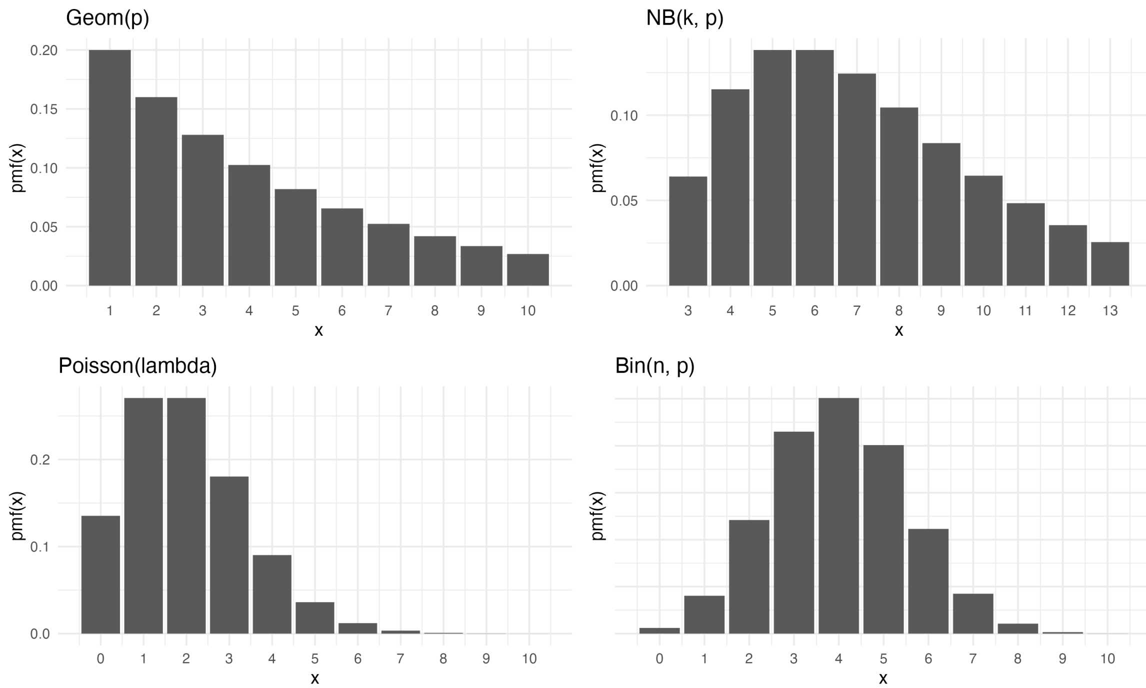
3. Let $X_1, X_2, \overset{\text{i.i.d.}}{\sim} Bernoulli(p)$ be an infinite sequence of independent random variables. We say that Y follows the *silly geometric* distribution if Y counts the number of X_i's that are zero before the first X_i that is a one. What are the pmf, expected value, and variance of Y?[2]

[2] For this and two following questions, the trick is to write the variable of interest in terms of a known distribution.

4. Let $X_1, \ldots, X_n$ a sequence of independent random variables where each X_i has a probability equal to $1/2$ of being $+1$ and probability of $1/2$ of being -1. Let W be the sum of the X_i's. What are the expected value and variance of W?

5. Let $X \sim Bin(n, p)$ and $Y = X/n$. Find the expected value and variance of Y. What are the limits of these two quantities as $n \to \infty$?

6. Four plots of probability mass functions are given on the following page. The Binomial shows the whole pmf; the others truncate the values for larger x. Estimate as best as possible the unknown parameters for each of the four distributions. Hint: You can use the range of the data to figure out one of the parameters for the two-parameter distributions. The mode is a good way to estimate the value of p for the Binomial. For the others, pick one value of the pmf and solve given the pmf formula.


Geom(p)
pmf(x)
0.00
0.05
0.10
0.15
0.20
1 2 3 4 5 6 7 8 9 10
x
NB(k, p)
pmf(x)
0.00
0.05
0.10
3 4 5 6 7 8 9 10 11 12 13
x
Poisson(lambda)
pmf(x)
0.0
0.1
0.2
0 1 2 3 4 5 6 7 8 9 10
x
Bin(n, p)
pmf(x)
0 1 2 3 4 5 6 7 8 9 10
x