Worksheet 12

1. Let $X \sim Bernoulli(p)$. Compute $m_X(t)$.

2. Let $X \sim Bernoulli(p)$. Using the value of $m_X(t)$, re-derive the expected value and variance for of X.

3. Describing $Y \sim Bin(n,p)$ as the sum of n random variables $X_1, \ldots, X_n \overset{\text{i.i.d.}}{\sim} Bin(1,p)$, determine the value of $m_Y(t)$. Hint: This should be easy.

4. Let $Y \sim Geom(p)$; we want to find $m_Y(t)$.[1] Start by writing down the definition of the mgf as a sum. We want to make the sum look like a geometry series (this is where the name of the distribution comes from). First, factor out a quantity of pe^t. Now, notice that you can write the remaining part as a sum of the form $\sum_{k=0}^{\infty} r^k$. This is a geometric series; when $|r| < 1$ the quantity converges and is equal to $\frac{1}{1-r}$. Use this to determine a closed form of $m_Y(t)$.

[1] This is a little harder, but I will break it down into smaller steps for you.

5. Let $Y \sim Geom(p)$. Using the mgf, what is $\mathbb{E}Y$? Hint: Use the chain rule and multiplication rule, not the division rule. This is a bit messy but there are no surprising tricks.

6. Let $Y \sim NB(k,p)$. What are $m_Y(t)$ and $\mathbb{E}Y$?

7. We want to compute the moment generating function for the Poisson distribution. To start, show that if $X \sim Poisson(\lambda)$, then:

$$m_X(t) = e^{-\lambda} \cdot \sum_{k=0}^{\infty} \frac{\lambda^k e^{tk}}{k!}$$

Then, show how to re-write this as:

$$m_X(t) = e^{-\lambda} e^{\lambda e^t} \cdot \sum_{k=0}^{\infty} \frac{(e^t \lambda)^k e^{-(e^t \lambda)}}{k!}$$

Set $\delta = e^t \lambda$ and notice that the value under the sum is a known quantity. Simply the result.

8. Let $X \sim Poisson(\lambda)$. What is $Var(X)$?

9. ($\star$) Think up a real-life, possibly very contrived, example of where you might see something that follows a Binomial, Bernoulli, Geometric, and Negative Binomial distribution in real life. Try to indicate what the parameters either are or try to approximate them.