Worksheet 07

1. The workers in a particular factory are 65% tall, 70% married, and 45% married and tall. If a worker is selected at random from this factory, find the probability that the worker is (a) a married non-tall person, (b) a single non-tall person, (c) married or tall or both.

2. Psychology majors are required to take two particular courses: PSYC 100 and PSYC 200. It is known that the chances of getting an A in PSYC 100 is .4 and the chances of getting an A in PSYC 200 is .3, while the chances of getting an A in both courses are .05. What are the chances that a randomly selected student will get at least one A in the two courses?

3. You give a friend a letter to mail. They forget to mail it with probability .2. Given that the letter is mailed it, the Post Office delivers it with probability .9. If the letter was not delivered, what is the probability that it was not mailed?

4. A student knows the answer to 40 of 50 multiple choice questions on an exam. They select an answer at random (from among 5 possible answers) for the remaining 10 questions. What is the probability that the student actually knew the answer to a particular question that they got correct?

5. We have a coin that comes up heads with probability 0.6. Assume that we flip the coin 5 times. Write H_j as the event that the j'th flip is heads. Assume that all of the events H_j are independent. What is the probability that the first 3 flips are heads and the next 2 flips are tails?

6. Continuing from the last question, notice that your answer is the probability of getting any sequence of five heads and tails where exactly three of them are heads.[1]. What, then, is the probability of getting exactly 3 heads in the sequence of 5 flips?

[1] So, it would apply for example to the probability of getting HTHTH and TTTHH as well

7. ($\star$) Consider a coin that is heads with probability p. Write down a general formula for the probability that there will be exactly k heads in a sequence of n flips using the coin.

8. Show that for arbitrary events A and B, we have:[2]

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B).$$

[2] For all of the remaining questions, start with the value on the left-hand side and proceed by applying steps based on our definitions that it can be manipulated to be equivalent to the right-hand side.

9. Show that if $A \subseteq B$, then $\mathbb{P}(B) \geq \mathbb{P}(A)$.

10. ($\star$) Show that if A and B are independent, then $\mathbb{P}(A|B^c) = \mathbb{P}(A)$.