Worksheet 02

1. Working in pairs (or triples if needed), select three marbles of one color and three marbles of another color. Pick one color to a reference color, which we will call C, and put all the marbles in a cup. Using the naïve definition of probability, what is the probability that a randomly selected marbel from the cup will be the color C?[1]

[1] I know the answer likely seems obvious, but try to actually write down the steps of the naïve definition.

2. The concept of the "probability of an event" can be defined as the proportion of times we would expect the event to occur as the outcome of a random "experiment" if the experiment is repeated a sufficently large number of times. We can use a simulation to approximate a probability using the empirical probability, defined as:

$$\text{empirical probability}\,(\text{event}) = \frac{\#\{\text{times event occured}\}}{\#\{\text{total trials}\}}$$

Note that this closely resembles the naïve definition of probability. Simulate the probability of choosing the color C from the cup with 12 random draws. How closely does your empirical probability match the one your calculated before?

3. Consider selecting one marble and then another from the cup, without replacing the first marble when selecting the second. Using the naïve definition of probability and the basic rule of counting, what is the probability that both marbles are the same color?[2]

[2] Do not resort to counting all the possibilites. Try to use the basic rule of counting.

4. Simulate the previous question with 12 random draws. Compare to the analytic solution.

5. Write down all of the possible outcomes of flipping a coin three times. Use H for heads and T for tails. What is the probability that all of the flips have the same result (in other words, 3 heads or 3 tails)?

6. Consider a set of cards with 3 colors/suits and 10 cards of each suit. If we select 5 cards from the deck, what's the probability that all of them are the same color/suit? This called a *flush* in poker. Note: Try to compute the numerical answer with a calculator.

7. Re-write your previous solution using only factorials.

8. Using the 26 letters in the latin alphabet, how many two-letter 'words' can you construct? These do not need to be actual words; just count the unique combinations.

9. How many combinations are there if every word needs a vowel (a, e, i, o, u)? How many combinations are there if we also allow words that end in y (like 'my')?

10. How many 'words' are there if we have three letters, without any vowel restrictions? How about four letters? What about words of length k?

11. $(\star)$[3] Consider a set of cards with 3 colors/suits and 10 cards of each suit. If we select $k+1$ cards from the deck for k between 1 and 9, write down the probability that all of the k cards have the same suit using factorials. Compute the probability for $k = 1, 2, 3, 4, 5, 6$.

[3] A star indicates that this will not appear as a normal exam question. It is usually for particularly involved questions that have a lot of parts.