Worksheet 06 (Solutions)

1. Let A, B, and C be three events in a random experiment with sample space S. Write expressions for each of the following sets in terms of the set operations "union," "intersection", "complement," and "difference":

(a) all three events occur
(b) A occurs but neither B nor C occur
(c) A and B occur but C does not occur
(d) at least one of the events occurs
(e) none of the events occur

Solution: These are examples of the solutions, though you may have other equivalent forms. For example, $A - B - C$ is equivalent to $A \cap B^c \cap C^c$.

(a) **all three events occur**: $A \cap B \cap C$
(b) **A occurs but neither B nor C occur**: $A - B - C$
(c) **A and B occur but C does not occur**: $A \cap B \cap C^c$ or $(A \cap B) - C$
(d) **at least one of the events occurs**: $A \cup B \cup C$
(e) **none of the events occur**: $A^c \cap B^c \cap C^c$ or $S - (A \cup B \cup C)$

2. Suppose $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and the sets A, B, and C are given by $A = \{2, 4, 6, 8, 10\}$, $B = \{2, 5, 6, 7, 10\}$, and $C = \{1, 6, 9\}$. Identify each of the following sets:[1]

[1] This one time only, I won't mind if you want to write directly on the sheet for this question.

(a) $A \cup B$
(b) $A \cap B$
(c) $A - B$
(d) $A \cup B^c$
(e) $A \cap B \cap C$
(f) $B \cap (A \cup C)^c$
(g) $(A \cap C) \cup (B \cap C)$
(h) $(A - C) \cup (C - A)$
(i) $A^c \cap B \cap C^c$

Solution:

(a) $A \cup B$ $\{2, 4, 5, 6, 7, 8, 10\}$

(b) $A \cap B$ $\{2, 6, 10\}$
(c) $A - B$ $\{4, 8\}$
(d) $A \cup B^c$ $\{1, 2, 3, 4, 6, 8, 9, 10\}$
(e) $A \cap B \cap C$ $\{6\}$
(f) $B \cap (A \cup C)^c$ $\{5, 7\}$
(g) $(A \cap C) \cup (B \cap C)$ $\{6\}$
(h) $(A - C) \cup (C - A)$ $\{1, 2, 4, 8, 9, 10\}$
(i) $A^c \cap B \cap C^c$ $\{5, 7\}$

3. Consider a cup with three marbles, two white and one black. The white marbles are indistinguishable. We want to model selecting a single marble from the cup. We will write the sample space for this experiment $\{w, b\}$ for a white and a black marble, respectively. Write down all of the possible events that can occur and what their probabilities are, assuming we are equally likely to pick each of the three marbles.

Solution: We have:

$$\begin{aligned} \mathbb{P}() &= 0 \\ \mathbb{P}(\{w\}) &= \frac{2}{3} \\ \mathbb{P}(\{b\}) &= \frac{1}{3} \\ \mathbb{P}(\{w, b\}) &= 1 \end{aligned}$$

4. The following table is a powerful tool for working through many classicial probability questions. For each of the rows, the first two columns add together to sum to the probability in the final column. For each of the columns, the first two rows sum up to the final row.

	$\boldsymbol{A}$	$\boldsymbol{A^c}$	
$\boldsymbol{B}$	$\mathbb{P}(A \cap B)$	$\mathbb{P}(A^c \cap B)$	$= \mathbb{P}(B)$
$\boldsymbol{B^c}$	$\mathbb{P}(A \cap B^c)$	$\mathbb{P}(A^c \cap B^c)$	$= \mathbb{P}(B^c)$
	$= \mathbb{P}(A)$	$= \mathbb{P}(A^c)$	$=1$

(a) Why must $\mathbb{P}(B) + \mathbb{P}(B^c)$ equal to 1? (b) Why must $\mathbb{P}(A \cap B) + \mathbb{P}(A \cap B^c)$ be equal to $\mathbb{P}(A)$? All of other relationships on the table have the same form these two values if you swap out the sets A and B for one another.

Solution: (a) The sets B and B^c are mutually exclusive since the second set is, by definition, everything not in B. Therefore:

$$\begin{aligned}\mathbb{P}(B) + \mathbb{P}(B^c) &= \mathbb{P}(B \cup B^c) \\ &= \mathbb{P}(S) \\ &= 1.\end{aligned}$$

(b) The sets $A \cap B$ and $A \cap B^c$ are also mutually exclusive. The first one can only have elements that are in B and the second one can only have elements not in B. Therefore, nothing can be in both sets. Therefore:

$$\begin{aligned}\mathbb{P}(A \cap B) + \mathbb{P}(A \cap B^c) &= \mathbb{P}((A \cap B) \cup (A \cap B^c))) \\ &= \mathbb{P}(A)\end{aligned}$$

The last line comes from seeing that if we take everything that is in either $A \cap B$ or $A \cap B^c$ is just everything in A (each element is either in the first of the second set).

5. Based on a (completely made-up) recent survey, it was determined that there is a probability of 0.4 that a randomly selected student's favorite fruit is an apple and a probability of 0.2 that a randomly selected student's favorite dessert is a brownie. It was also determined that there is a probability of 0.1 that randomly selected student reported their favorite fruit as an apple and their favorite dessert to be a brownie. (a) What is the probability that a randomly selected selected student's favorite fruit is not an apple? (b) What is the probability that a randomly selected student's favorite dessert is not a brownie? (c) What is the probability that a randomly selected student has a favorite fruit that is not an apple and a favorite dessert that is not a brownie. Hint: Define A to be the event that their favorite fruit is an apple, B the event that their favorite dessert is a brownie. Then, just fill in the table in the previous question.

Solution: The following table shows all of the values, with the directly given ones in bold.

	$\boldsymbol{A}$	$\boldsymbol{A^c}$	
$\boldsymbol{B}$	**0.1**	0.1	=**0.2**
$\boldsymbol{B^c}$	0.3	0.5	=0.8
	=**0.4**	=0.6	=**1**

Reading off the table, the answers are: (a) 0.6, (b) 0.8, and (c) 0.5.