Handout 11: Moments

Let X be a random variable. The (raw) **k'th moment**, for any positive integer k is defined as:

$$\mu'_k = \mathbb{E}\left[X^k\right]$$

The **k'th centered moment** is given by:

$$\mu_k = \mathbb{E}\left[(X - \mathbb{E}X)^k\right].$$

The 2nd centered moment is also called the **variance** of a random variable.[1] It describes how spread out the random variable is away from its mean. We have the following notation and special form of the variance:

$$\begin{aligned} Var[X] &= \mathbb{E}\left[[X - \mathbb{E}[X]]^2\right] \\ &= \mathbb{E}\left[X^2 + \mathbb{E}[X]^2 - 2X \cdot \mathbb{E}[X]\right] \\ &= \mathbb{E}[X^2] + \mathbb{E}[X]^2 - 2 \cdot \mathbb{E}[X]^2 \\ &= \mathbb{E}[X^2] - \mathbb{E}[X]^2 \\ &= \mu'_2 - (\mu'_1)^2 \end{aligned}$$

[1] You have probably heard of the variance of a set of numbers. This value, called the sample variance to distinguish it from the formula here, should also converge to the theoretical variance if you repeat a random experiment a large number of times.

For any constant $\alpha \in \mathbb{R}$, we have:

$$Var[\alpha \cdot X] = \alpha^2 \cdot Var[X]$$

And when X and Y are independent random variables, we have:

$$Var[X + Y] = Var[X] + Var[Y]$$

We will not really need the other moments for practical applications, but will use them for establishing theoretical results such as the central limit theorem.