Handout 08: Law of Total Probability

Let $C_1, \ldots, C_k$ be a partition of the sample space S into k mutually exclusive sets. In other words, we have $C_i \cap C_j = \emptyset$ for all $i \neq j$ and $C_1 \cup \cdots \cup C_k = S$. Then, for any set A, we have by our definitions of probabilility and conditional probability:

$$\begin{aligned}\mathbb{P}[A] &= \mathbb{P}[C_1] \cdot \mathbb{P}[A|C_1] + \cdots + \mathbb{P}[C_k] \cdot \mathbb{P}[A|C_k] \\ &= \sum_k \mathbb{P}[C_k] \cdot \mathbb{P}[A|C_k].\end{aligned}$$

In class, I showed the version of this with $k = 3$, and we had previously used the result with $k = 2$. This result is called the **Law of Total Probability (LOTP)**, and is a surprisingly powerful method for solving all kinds of probability questions.