Common Distributions and Inequalities

The following table provides the notation, density, mean, variance, and moment generating functions of the families of distributions that we will study. A few of the more complicated formulae are left blank. I have not written the support of the parameters of density functions; these should be clear from the way we defined them. Note that the Geometric, Gamma, and Negative Binomial have alternative parameterizations. You may see different results in other resources.

Distribution	**Notation**	**PMF/PDF**	**Mean**	**Variance**	**MGF**
Bernoulli	$Bernoulli(p)$	$p^x(1-p)^{1-x}$	p	$p(1-p)$	$(1-p+pe^t)$
Binomial	$Bin(n,p)$	$\binom{n}{x}p^x(1-p)^{n-x}$	np	$np(1-p)$	$(1-p+pe^t)^n$
Geometric	$Geom(p)$	$(1-p)^{x-1}p$	$1/p$	$(1-p)/p^2$	$\frac{pe^t}{1-(1-p)e^t}$
Negative Binomial	$NB(k,p)$	$\binom{x-1}{k-1}(1-p)^{x-k}p^k$	k/p	$k(1-p)/p^2$	$\left(\frac{pe^t}{1-(1-p)e^t}\right)^k$
Hypergeometric	$HG(N,K,n)$	$\binom{K}{k}\binom{N-K}{n-k}/\binom{N}{n}$	nK/N	·	·
Poisson	$Poisson(\lambda)$	$\frac{\lambda^x e^{-\lambda}}{x!}$	λ	λ	$e^{\lambda(e^t-1)}$
Exponential	$Exp(\lambda)$	$\lambda e^{-\lambda x}$	λ^{-1}	λ^{-2}	$\frac{\lambda}{\lambda-t}$
Gamma	$Gamma(\alpha,\beta)$	$\frac{1}{\Gamma(\alpha)\beta^\alpha}\cdot x^{\alpha-1}e^{-x/\beta}$	$\alpha\beta$	$\alpha\beta^2$	$(1-\beta t)^{-\alpha}$
Beta	$Beta(\alpha,\beta)$	$\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\cdot x^{\alpha-1}(1-x)^{\beta-1}$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	·
Uniform	$U(a,b)$	$(b-a)^{-1}$	$\frac{1}{2}(b-a)$	$\frac{1}{12}(b-a)^2$	·
Normal	$N(\mu,\sigma^2)$	$\frac{1}{\sqrt{2\pi\sigma^2}}e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$	μ	σ^2	$\exp\{\mu t+\frac{1}{2}\sigma^2t^2\}$
Chi-squared	χ^2_k	$\frac{1}{2^{k/2}\Gamma(k/2)}\ x^{k/2-1}e^{-x/2}$	k	$2k$	$(1-2t)^{-k/2}$
Student-T	t_n	·	0	t / (t - 2)	·
F-Distribution	$F(d_1,d_2)$	·	$\frac{d_2}{d_2-1}$	·	·
Cauchy	$C(\gamma)$	$\frac{1}{\pi/\gamma(1+(x/\gamma)^2)}$	undefined	undefined	undefined

We will also find it useful to have a reference for the following inequalities.

(Markov's Inequality) Let X be a random variable. Then for any $a>0$,

$$\mathbb{P}[|X|\geq a]\leq\frac{\mathbb{E}|X|}{a}.$$

(Chebyshev's Inequality) Let X be a random variable with mean μ and variance σ^2. Then for any $a>0$,

$$\mathbb{P}[|X-\mu|\geq a]\leq\frac{\sigma^2}{a^2}.$$

(Chernoff's Inequality) Let X be a random variable. Then for any $a>0$ and $t>0$,

$$\mathbb{P}[X\geq a]\leq\frac{\mathbb{E}[e^{tX}]}{e^{ta}}.$$